Closing tonight: 2.7
Closing Mon: 2.7-8
Closing Wed: 2.8
Closing Fri: 3.1-2
Visit office hours 1:15-3:30pm in Com B-006

2.7-8 Derivatives Intro

Summary: Given $y = f(x)$, we were trying to find <u>the slope of the tangent line</u> at the point $(x_1, y_1) = (a, f(a))$.
We took a "nearby" second point $(x_2, y_2) = \left(a + h, f(a + h)\right)$.

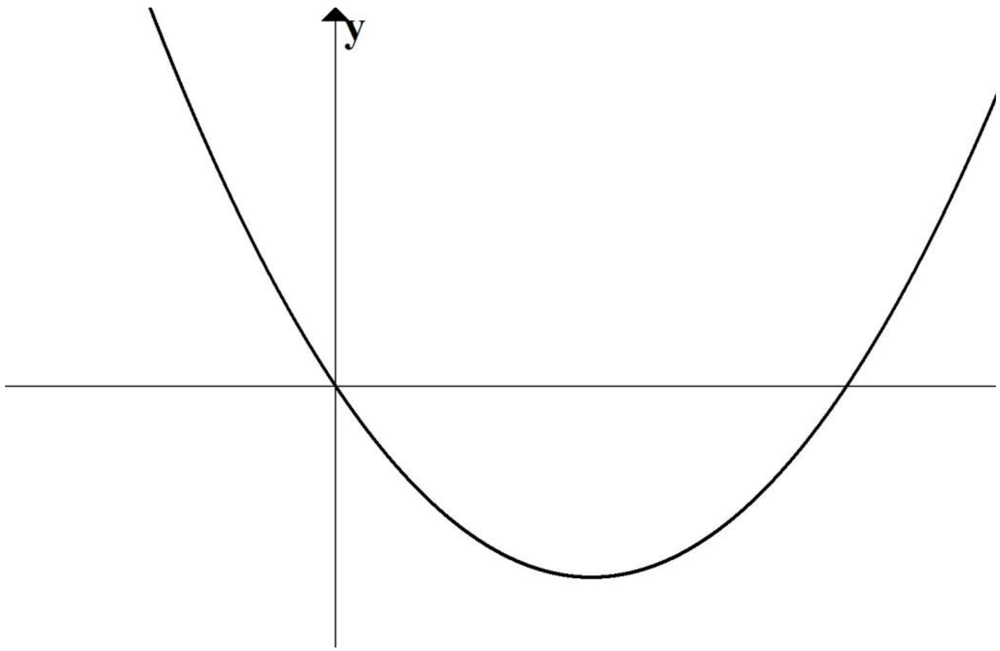
Slope of *secant* = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{f(a+h) - f(a)}{a+h-a}$

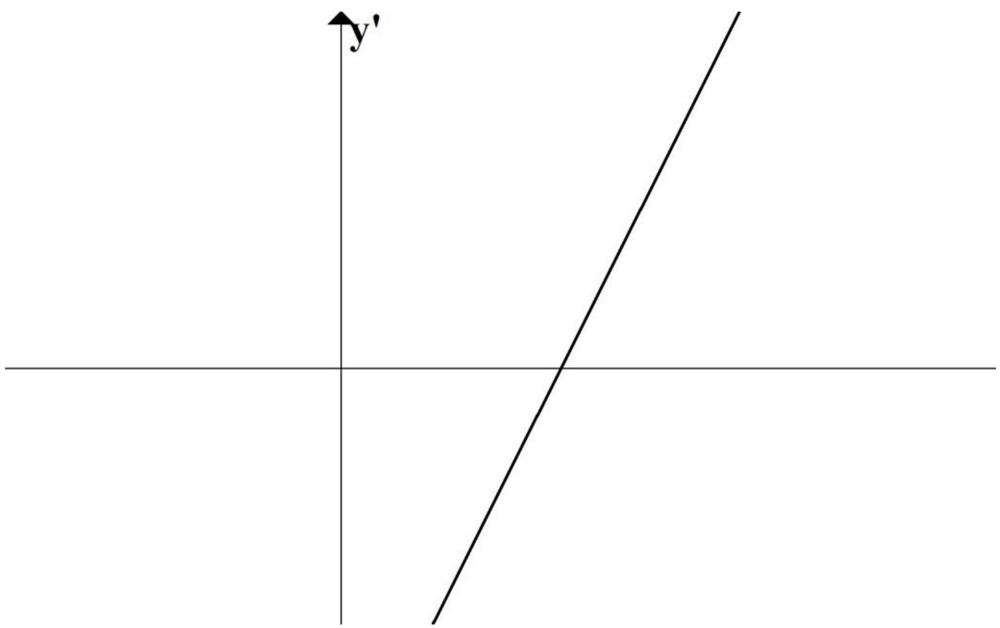
Thus, we defined the derivative (*i.e.* slope of tangent) by

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

Entry Task: $f(x) = 2x^2 - 3x$

1. Find $f'(4)$.
2. Give the equation of the tangent line at $x = 4$.
3. Find $f'(x)$.

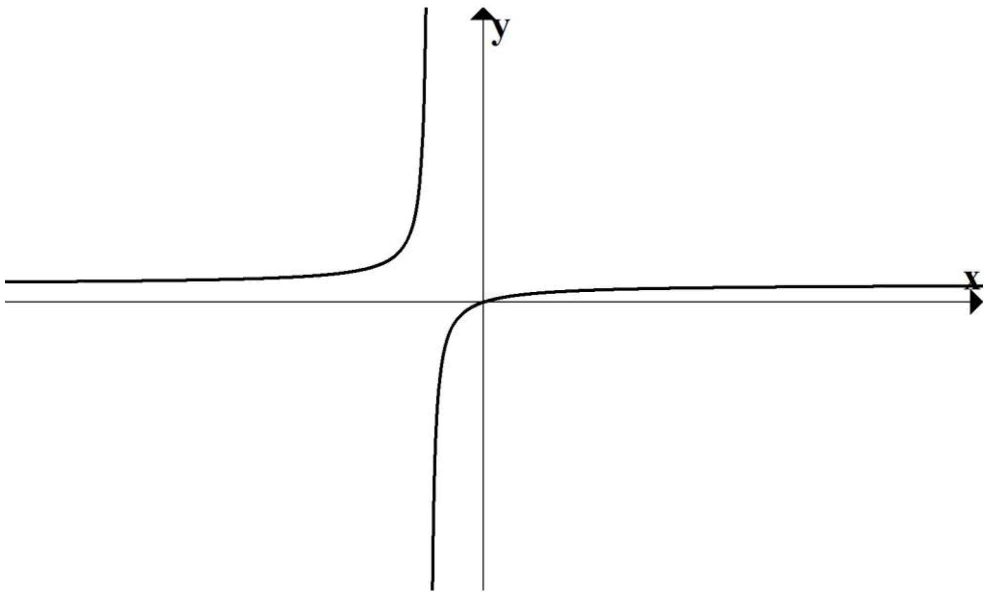

y

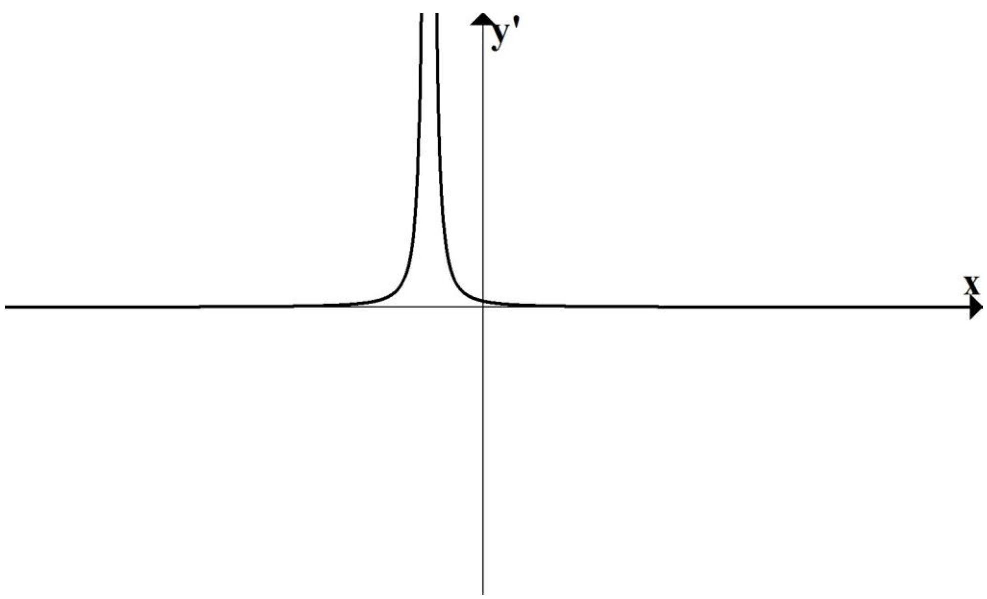

y'

Example:

$$g(x) = \frac{2x}{x+3}$$

1. Find $g'(2)$.
2. Give the equation of the tangent line at $x = 2$.
3. Find $g'(x)$.


y
x


y'
x

Observations:

Given $y = f(x)$.

- $y = f'(x)$ is a new function.
- $f(x) =$ "height of the graph at *x*"
- $f'(x) =$ "slope of the tangent to $f(x)$ at x"
- We call it the "instantaneous rate of change" (speedometer speed)
- The units of $f'(x)$ are $\frac{\text{y–units}}{\text{x–units}}$.

Fundamental to all applications:

$y = f(x)$	$y = f'(x)$
horiz. tangent	zero
increasing	positive
decreasing	negative

Notation:

Early we found

if $f(x) = 2x^2 - 3x$,

then $f'(x) = 4x - 3$.

Other ways to write this include:

$$y' = 4x - 3$$

$$\frac{dy}{dx} = 4x - 3$$

$$\frac{d}{dx}(2x^2 - 3x) = 4x - 3.$$

Later we will also discuss:

$$f''(x) = y'' = \frac{d(dy/dx)}{dx} = \frac{d^2y}{dx^2}$$

Example:

if $y = f(x) = 2x^2 - 3x$,

then $y' = f'(x) = 4x - 3$

and $y'' = f''(x) = 4$

which can also be written as

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(4x - 3) = 4$$

Differentiability

Sometimes we can have a place where "slope of tangent" doesn't make sense.

Definition: We say a function, $y = f(x)$ is **<u>differentiable</u>** at $x = a$ if the following limit exists:

$$\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

Otherwise it is not differentiable at $x = a$.

In order to get differentiable:

1. It must be defined at x = a.
2. It must be continuous at x = a.
3. The "slope" must be the same from both sides.